Lesson Research Proposal for Grade 7 Combining Like Terms

For the lesson on combining like terms
At Brentano Math and Science Academy, Mr. Bingea's class
Instructor: Aaron Bingea
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1. Creating an Argument for Combining Like Terms

2. Brief description of the lesson

This lesson will give students several opportunities to develop an argument for combining like terms. The central problem involves a pool being filled up with water by multiple hoses that fill at different rates. Students will be asked to explain how they calculated the amount of water in the pool after a given number of minutes and eventually pushed to generating an expression to model the amount of water in the pool after x minutes. We expect students will naturally start to combine like terms after multiple iterations of this problem and will be able to create an argument as to why certain terms can or cannot be combined by using the problem context and the distributive property to justify.

3. Research Theme

The goal for this lesson is for students to develop their proficiency with the Standard of Mathematical Practice Three, construct viable arguments and critique the reasoning of others. We want to see students reaching mathematical conclusions about what terms in an algebraic expression may be combined using strategies of factoring and expanding to justify their arguments and negate the arguments of others.

4. Goals of the Unit

a) Students will produce equivalent expressions using their knowledge of operations, factoring, and the distributive property. They will justify their equivalence with an argument using diagrams and/or language that can be conveyed to classmates.
b) This mini-unit is the culmination of their larger expressions, equations, and inequalities unit. In this unit they represent relationships of two quantities with tape diagrams and with equations, and explain correspondences between the two types of representations; solve equations of the forms $px+q=r$ and $p(x+q)=r$, then solve problems that can be represented by such equations; and solve inequalities that represent real-world situations.

5. Goals of the Lesson:

a) Students will understand why like terms can be combined to create an equivalent expression
b) Students will understand why a term with a variable cannot be combined with a constant

6. Relationship of the Unit to the Standards

In this unit, students work with equivalent linear expressions, using properties of operations to form an argument to explain equivalence (SMP 3). They represent expressions with area diagrams, and use the distributive property to justify factoring or expanding an expression.

7.

Related prior learning standards	Learning standards for this unit	Related later learning standards
6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* <u>6.EE.A.4</u> Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for..* <u>6.EE.A.2.c</u>	<u>7.NS.A.1</u> Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. <u>7.EE.A.1</u> Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. <u>7.EE.B.4.A</u> Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*	HS.A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* HS.F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2*.		

8. **Background and Rationale**

Topic

Our team agreed that our previous treatment of the skills and concepts involved with combining like terms has been mostly abstract and procedural. In past years we feel that our students know the concept of combining like terms as simply collecting all the terms that look similar. Our current 8th graders know without justification that you can put x's with x's, y's with y's, and numbers with numbers. Our goal is to remedy this surface level knowledge for our current 7th grade students and have them develop the mathematical reasoning as to why we can create equivalent expressions by combining like terms.

Context

In our research, we found that this topic was only introduced in the abstract. Students are typically given expressions and then are tasked with combining like terms. The purpose of our lesson is for students not only to perform the skills involved with combining like terms but to more importantly, justify why terms are allowed to be combined. We are choosing to connect the expressions to the real-life context of filling up a pool with water so that the quantities have concrete meaning and aid our students ability to justify why certain terms can or cannot be combined. By tasking the students to reason with a concrete problem first we predict that students will be more prepared create an argument and justify why we can combine like terms in the abstract. As a result, we specifically chose simple numbers to represent the quantities having three be the water already in the pool and four and five the respect rates of the hoses filling up the pool so students would not struggle with the computation. As the lesson progresses, we make the numbers more challenging having 40 represent the amount of water already in the pool and 37 and 13 representing the rate of the hoses. In this context, we want the computation to be more complicated forcing students to apply strategies of combining like terms and factoring to get a solution.

Student Discussion

Our students are familiar with the context of filling a vessel with water. It has been used previously in our unit on integer operations. As a result, we predict that students will be able to comfortably articulate their case and respond to their classmates. Throughout the lesson students will be given opportunities to first discuss their ideas with their partners before bringing their ideas to the whole group discussion. This gives students the opportunity to make sense of and test their argument before hearing ideas from the broader class.

8. Research and Kyozaikenkyu

To begin researching for this lesson, the team began to unpack the standard and look at the buildup of teaching the distributive property as it relates to combining like terms between 6th and 7th grade. In 6th grade in both Engage NY and Illustrative Mathematics students generate equivalent expressions by using area models and order of operations to apply the distributive property through factoring and expanding of non-negative whole numbers--2(3+8x)= 6+16x. As students progress to 7th grade the curricula shifts to problems where students encounter linear expressions involving more operations and rational numbers, requiring an understanding of multiplying with negative numbers such as 7-2(3-8x).

In Engage NY, students begin the unit by writing equivalent expressions by finding sums and differences applying both the commutative and associative property to collect like terms and rewrite algebraic expressions in standard form. From there students progress to rewriting products in standard by applying the commutative property to rearrange like terms--numeric coefficients, like variables--next to each other. Students rewrite division as multiplying by the multiplicative inverse. In the following two lessons students use area models and the distributive property to first multiply one term by a sum of two or more terms to expand a product to a sum and then reverse the process to rewrite the sum as a product of the greatest common factor and a remaining factor. Once the students have these prerequisite skills, they model problems with expressions in both forms--factored form and expanded form--to see how the quantities are related.

Illustrative Mathematics begins the unit with students using graphic organizers to work with the distributive property. They learn how to rewrite subtraction as adding the opposite in order to use the commutative property. From there, students apply the distributive property to expand and factor linear expressions with rational coefficients. In the next lesson, students then begin to find an expression that, when combined with another expression, yields an equivalent expression. They apply properties of operations to generate an equivalent expression with fewer terms. Once students are familiar with factoring and expanding to combine like terms, they identify and correct errors made when applying properties of operations (See problem set progression below).

5. a. Expand to write an equivalent expression: $\frac{-1}{4}(-8x + 12y)$

   b. Factor to write an equivalent expression: $36a - 16$

6. Tyler is simplifying the expression $6 - 2x + 5 + 4x$. Here is his work:

$$6 - 2x + 5 + 4x$$

$$(6 - 2)x + (5 + 4)x$$

$$4x + 9x$$

$$13x$$

   a. Tyler's work is incorrect. Explain the error he made.

   b. Simplify the expression $6 - 2x + 5 + 4x$.

The unit concludes with students generating a variety of expressions by positioning parentheses in different places within an expression and then applying properties to write the expressions with fewer terms.

9. Unit Plan

Lesson	Learning goal (s) and tasks	Problem
1	Students will recall the distributive property from 6th grade Use a graphic organizer for work with the distributive property. Understand how to rewrite subtraction as adding the opposite in order to use the commutative property.	Select **all** the expressions that are equivalent to $4 - x$. a. $x - 4$ b. $4 + -x$ c. $-x + 4$ d. $-4 + x$ e. $4 + x$ Use the distributive property to write an expression that is equivalent to $5(-2x - 3)$. If you get stuck, use the boxes to help organize your work.

2	Apply the distributive property to expand and factor linear expressions with rational coefficients	. Expand to write an equivalent expression: $-\frac{1}{2}(-2x + 4y)$. Factor to write an equivalent expression: $26a - 10$
3		
4 Research Lesson	Students will understand why like terms can be combined to create an equivalent expression Students will understand why a term with a variable cannot be combined with a constant	A pool starts with 4 gallons of water. Two hoses are turned on and begin filling up the pool. Hose A fills up the pool at a rate of 2 gallons per minute. Hose B fills up the pool at a rate of 3 gallons per minute. Task: Generate an expression to represent how much water is in the pool after x minutes. A swimming pool starts with 40 gallons of water. Two hoses are turned on and begin filling up the pool. Hose A fills up the pool at a rate of 37 gallons per minute. Hose B fills up the pool at a rate of 13 gallons per minute. There is also a leak in the pool, and it LOSES 10 gallons per minute. Task: Generate an expression to represent how much water is in the pool after x minutes.
5	Apply all properties of operations to generate an equivalent expression with fewer terms.	Some students are trying to write an expression with fewer terms that is equivalent to $8 - 3(4 - 9x)$. Noah says, "I worked the problem from left to right and ended up with $20 - 45x$." $8 - 3(4 - 9x)$ $5(4 - 9x)$ $20 - 45x$ Lin says, "I started inside the parentheses and ended up with $23x$." $8 - 3(4 - 9x)$ $8 - 3(-5x)$ $8 + 15x$ $23x$ Jada says, "I used the distributive property and ended up with $27x - 4$." $8 - 3(4 - 9x)$ $8 - (12 - 27x)$ $8 - 12 - (-27x)$ $27x - 4$ Andre says, "I also used the distributive property, but I ended up with $-4 - 27x$." $8 - 3(4 - 9x)$ $8 - 12 - 27x$ $-4 - 27x$

10. Research lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction A swimming pool starts with 3 gallons of water. Two different hoses are turned on and begin filling up the pool. The first hose fills up the pool at a rate of 2 gallons per minute. The second hose fills up the pool at a rate of 4 gallons per minute. How much water is in the pool after 5 minutes? T: Take some time to solve this in your notebooks. Show the calculations you used to find the answer. Draw a picture if it would help. Students work independently for 3 minutes Expected student responses: 3 + 2(5) +4(5) = 33 3 + 6(5) = 33 *9(5) = 45 (misconception)*	Teacher will display a visual of the scenario. Visual and problem statement will be in the notebooks. Teacher will prompt students, as needed: -Label the visual -How much water would there be after one minute? -How much water would there be if it were just one hose?	Are students accounting for all terms? Do students understand the context? Are students using different strategies to highlight?
------------------ Discussion Highlight correct response- Ask them what every term means? Highlight the combined version- Ask them what this student did? Misconception- Combining the starting value Ask students to explain what x student may have been thinking. Do you agree?	--------------------------- Boardwork and labelling all parts of student work tying it back into context	------------------------ Are students engaged in discussion? Are students asking clarifying questions? Are students in agreement?
----------------- How much water is in the pool after **7** minutes? T: Take some time to solve this in your notebooks. Show the calculations you used to find the answer. Draw a picture if it would help. Students work independently for 3 minutes	-------------------------------------- Use of board work to highlight connection in responses.	------------------------ Are students using their work from the first problem and substituting the number of minutes into their old work.

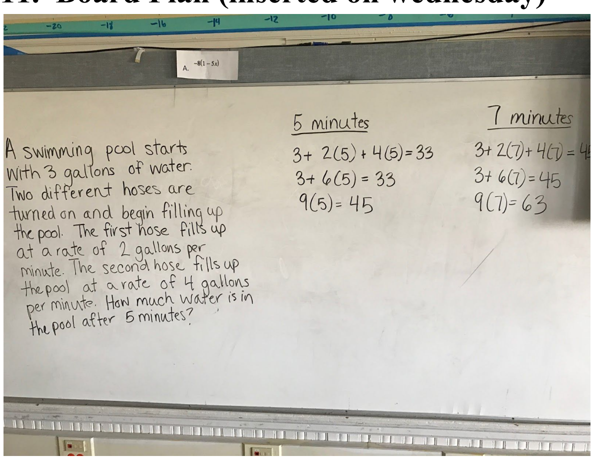
Expected student responses: 3 + 2(7) +4(7) = 45 3 + 6(7) = 45 *9(7) = 63 (misconception)* Turn and talk: What is the same/different about your work on these two problems? Discussion: Highlight that only the number of minutes changes. -- **Posing the Task** T: How much water is in the pool after *x* minutes? Generate an expression to represent the number of gallons in the pool after x minutes. Students independently generate expressions to represent the situation. Present the anticipated responses: A. 3 + 2x + 4x B. 3 + 6x C. 9x Prompt students to turn and talk- Which expression(s) do you agree with and why? Questions to discuss whole group- Does expression A match this situation? Solidify this first so that B and C can be discussed in comparison to A. After student responses, teacher will present 3 + ( )x and ask where the ‘6’ came from. Is expression B equivalent to expresion A? How do we know? Is expression C equivalent to B/A? How do we know?	-- Responses are written on the board.	Has the misconception been cleared up? Do students notice that only the number of minutes changes? ------------------------- Are students generating the anticipated responses? Are students using the problem context to justify why their expressions are equivalent? Are students able to explain why terms accurately model the problem?

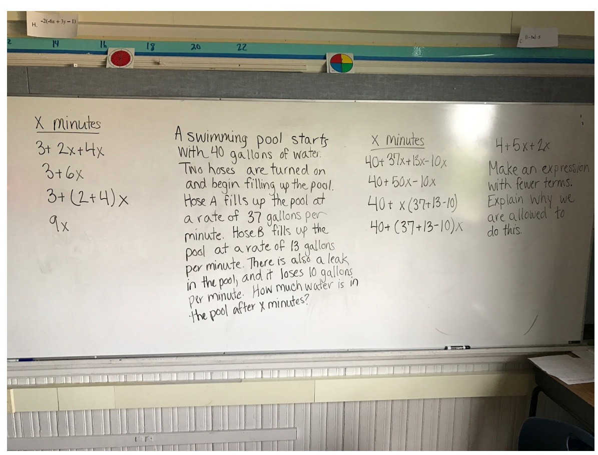
Why does expression C not work? Why can’t we just combine all the numbers? ------------------- Present task 2: A swimming pool starts with 40 gallons of water. Two hoses are turned on and begin filling up the pool. Hose A fills up the pool at a rate of 37 gallons per minute. Hose B fills up the pool at a rate of 13 gallons per minute. There is also a leak in the pool, and it LOSES 10 gallons per minute. How much water is in the pool after x minutes? T: Take some time to solve this in your notebooks. Show the calculations you used to find the answer. Draw a picture if it would help. Students work independently for 3 minutes Present the anticipated responses: A. 40 + 37x + 13x -10x B. 40 + 50x - 10x C. 40 + 60x D. 40 + 40x E. 40 + x (37 + 13 - 10) or 40 + (37 + 13 - 10) x F. 80x **Questions to discuss whole group-** Clear up any misconceptions, by having students look for any errors. Students will address these errors by talking to their table partners and then clearing. How do we know A and B are a equivalent?	------------------------------ A new visual for this context is presented.	-------------------------- Are students combining like terms? Are students factoring out the x? Are student’s correctly representing leak as -10x? Are students

Why are A and D equivalent? Using your same argument. Why can't we combine all of terms in the expression? Why can't we combine the 40 with the 37 and the 13?		
Summing up 4 + 5x + 2x Make an equivalent expression with fewer terms. Explain why we are allowed to do this.	Prompt is written on the board.	Are students simplifying the expression accurately? Are students justifying why the like terms can be combined?

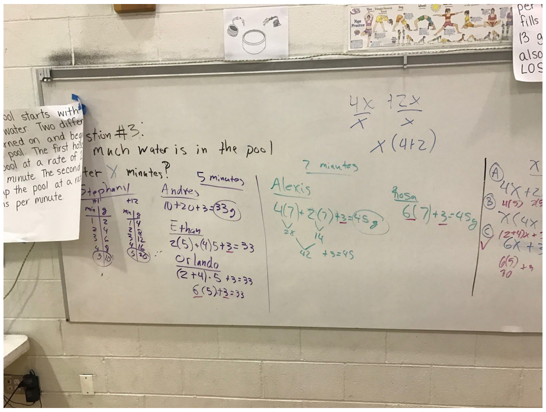
10. **Evaluation**
    - Did the lesson successfully promote students to construct a viable argument as to why like terms can be combined?
    - Did students understand why like terms can be combined to create an equivalent expression
    - Did students why a term with a variable cannot be combined with a constant

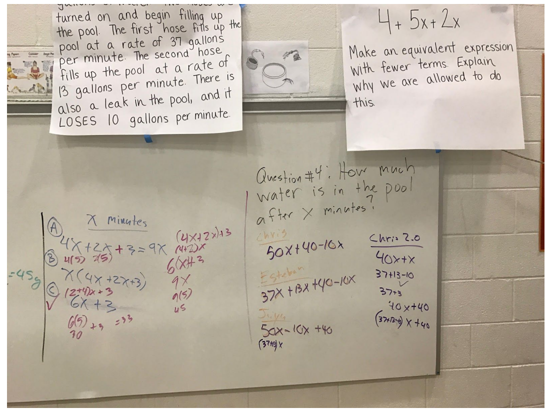
11. **Board Plan (inserted on wednesday)**





12. Reflection





Our team was satisfied with the students responses and discussion that occurred during the lesson. The lesson proved to elicit the desired misconceptions and varying strategies to combine like terms. The majority of students used the context of filling up a pool with water to justify why certain terms could or couldn’t be combined. We also felt that students were using each other’s arguments to refine their own during turn and talks and in whole group discussions. Multiple times throughout the lesson students referred to an idea of another classmate’s to make a point or explain their answer.

Overall we thought the lesson fell short in meeting our most advanced goal of having students justify combining like terms using the distributive property. Many students were able to clearly state why we can combine the rates of each hose because they were both being multiplied by minutes. However we did not see any evidence that students abstracted this idea to the point where students were combining like terms by factoring out the variable. After reflecting on this fact and the post-lesson discussion where this was debated, we concluded that this was an issue with the lesson goal and not the lesson itself. Because this was the students’ first experience with combining like terms, our goal should have focused more on using the context to justify why we are allowed to combine like terms and why it can be useful. We felt this goal was largely achieved and properly set students up to address the more abstract justification using the distributive property in the next lesson.

Another significant takeaway for our students was their reliance of the problem context when discussing whether or not different expressions were equivalent or not. In our research we could not find a curriculum that treated this topic first with a concrete problem solving context. Instead most units taught combining like terms with algebraic expression void of any context. We felt that this lesson proved that students benefited from making connections to the different terms with a relevant real-world situation and allowed most students to construct a sound mathematical justification for combining like terms. We have traditionally taught this skill briefly in the abstract without giving students time to develop a robust justification for the skill. The success of this lesson in this regard has led us to consider more algebraic skills that could be taught first with problem contexts so that student could develop a more conceptual justification as to why certain algebraic moves can be made.

In the final comments, the issue of the discussion moving too far away from the context was raised. The team disagreed with this assessment to a certain extent. Students were clearly using the problem context to discuss why they agreed or disagreed with different expressions. IN partners and in whole group we heard students talking about gallons per minute, how the constant represented the starting value, and substituting different values for the variable to see if the expressions were equivalent for a given number of minutes. However we did agree that this wasn't reflected in the board work. To keep a stronger focus on the context throughout the discussion, we could have labeled students responses with units and even labeled the different numbers in each each expression with what they meant in terms of the problem. Even though the students were making these connections verbally, we agree that students would have benefited from seeing it on the board as well.

Another issue raised during the post-lesson discussion was whether or not the lesson should have moved from tasking students with finding how much water was in the pool after 5 minutes, 7 minutes and then "x" minutes or in reverse order. In the reverse order, we would have first asked students to find how much water was in the pool after x minutes and they could have used the examples of 5 or 7 minutes to justify why an expression would work or not. Ultimately we felt that we would keep the format of the lesson the same. Because we started with giving students a more concrete task of generating an expression for 5 and 7 minutes, they were able to make connections with strategies and see how some students were combining the rates of the hoses in the same way from problem to problem. We saw students ultimately rely on these concrete iterations of the problem to grapple with the abstract task of finding how much water was in the pool after x minutes. Even though the number of students successfully completing the task independently decreased from the first problem to the third problem, the number of students that were able to engage in the argument around which expressions were equivalent and why we could combine the rates of the hoses increased. Their arguments built intuitively with very little teacher input. We saw this as a result of concrete to abstract progression of the lesson.